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MINIMIZATION OF MUSCLE FATIGUE AS THE CRITERION TO SOLVE MUSCLE FORCES-SHARING PROBLEM

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ABSTRACT

The application of functional electrical stimulation (FES) to muscles quickly fatigues them. Our research goal is to determine the optimal control of FES signals that delay the fatigue for as long as possible. In this research we have used a physiology-based mathematical model of muscle fatigue, to study the behaviour of a musculoskeletal system during a prolonged exercise. To solve the redundant problem of muscle force sharing, we have used a time-dependent fatigue minimization objective instead of the usual activation-based minimization criteria. Our results showed that muscle co-activation, as seen in natural human motion, does not necessarily minimize muscle fatigue.

NOMENCLATURE

α_i	Fourier Coefficient
β_i	Fourier Coefficient
θ	Flexion angle
A	Non linear activation coefficient
a	Muscle activation level
B	Non linear activation coefficient
C	Muscle fibre transition drive
d	Centre of mass distance from elbow
F	Muscle force
f_l	Muscle force-length relation
f_v	Muscle force-velocity relation
$F_{0_{max}}$	Relaxation rate
F_{max}	Maximum isometric muscle force
I	Forearm moment of inertia
k_f	Fatigue rate
k_r	Recovery rate
L_d	Development rate
L_r	Relaxation rate
M_a	Active muscle fibres
M_f	Fatigued muscle fibres
M_r	Resting muscle fibres
p	Exponent in the cost function
R	Muscle moment arm
w	Total of forearm, hand, and load weights

INTRODUCTION

Locomotion has been one of the key elements in human evolution since early times. Alongside the development of limbs and musculoskeletal systems, the control mechanism is also evolved. The human body is a complex multi-degree of freedom system, actuated by a large number of muscles. The central nervous system (CNS) controls this complex system with such an ease that we hardly notice the challenges. The human actions are remarkably adaptive, efficient and robust.

From a mechanical point of view, the human body has more than the minimum number of muscles required to move a joint. Therefore, to reach a unique solution for the redundant problem of muscle force-sharing in the human musculoskeletal system, extra criteria need to be considered. The efficiency of mo-

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tions has been the motivation for researchers to seek optimization methods to calculate muscle activities during a motor action. Minimizing an index of effort is the usual practice to pick the best solution. Various indices are proposed in the literature to represent the muscular effort, including muscle force [1,2], muscle activation level [3–7], and physiological energy consumption [4,5,8,9].

Muscle fatigue can also be used as the index to solve for the unique muscle activities. To the best of our knowledge, minimization of a physiological muscle fatigue index (as opposed to the activation-based interpretation of muscle effort) has not been preformed.

Our major motivation for choosing muscle fatigue as the minimization criterion is the rapid occurrence of fatigue during application of external electrical pulses to the muscles via functional electrical stimulation (FES). In FES, electrodes are used to inject electrical current to the muscles, which artificially depolarizes the muscle fibre cells, and causes the muscle to produce force. FES has been shown to be an effective rehabilitation method [10, 11]. However, its major drawback is the unnatural recruitment of motor units [12], which expedites muscle fatigue. Therefore, finding muscle activation patterns that minimize muscle fatigue can have important implications in the optimal control of FES devices.

To study muscle fatigue, we have used an efficient mathematical model of fatigue [13] in our simulations of musculoskeletal systems. This model has the advantage of being simple (unlike the more complex models such as [14] or [15]), while capturing the physiological process of fatigue. Simpler fatigue models such as [16] and [17] seem to be useful in ergonomics studies, but lack the fidelity required in predictive musculoskeletal models. In this fatigue model, the pool of muscle fibres is divided into three compartments: the active fibres (generating force), the rested fibres (not generating force, but are ready to be recruited), and the fatigued fibres (cannot generate force until recovered). When the muscle is activated, its active fibres will eventually become fatigued (move to the fatigued compartment); therefore, our objective is to minimize the number of muscle fibres in the fatigued compartment.

Since this representation of muscle fatigue is time-dependent, static optimization [1, 18] is not suitable. Thus, a dynamic optimization approach is needed to account for changes of muscle fatigue during the entire exercise/simulation period. Since muscle fatigue is a relatively slow process, long simulation times are necessary. Therefore, it is infeasible to solve for all muscle excitation signals at each time-step. To overcome this challenge, a parametrized signal [5] can be used.

The scenario we have studied in our simulations is the stationary holding of a weight in hand for a long time. We have used a mixture of forward and inverse dynamic simulations. Since the arm is not moving, we have assumed constant joint angles and zero velocities, and treated the acceleration as a constraint (has to be zero). The fatigue model, on the other hand, is a forward dynamic simulation, in which the differential equations of fatigue development are integrated over time. This approach results in faster simulations, without sacrificing the accuracy.

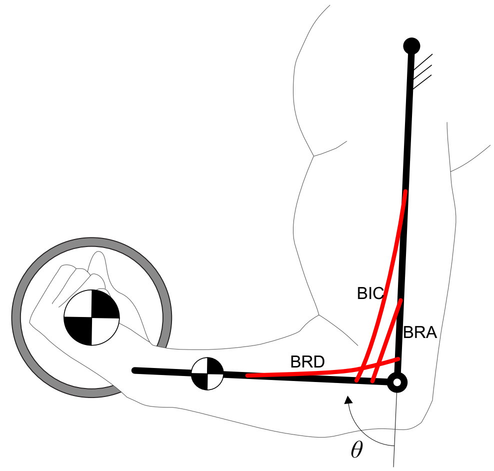

FIGURE 1. THE MUSCULOSKELETAL FOREARM MODEL

To summarize our methodology, we have applied our fatigue-minimization framework to a one degree of freedom musculoskeletal model of human forearm. The details of the musculoskeletal model and the fatigue dynamics are provided in the next section. We have then used a global parametrization approach to find the muscle excitation patterns that minimize the fatigue, while keeping the arm stationary.

METHODS

Musculoskeletal Modelling

A one-degree-of-freedom model of a human arm is considered in this work (Fig. 1). In this model, the forearm is assumed to be stationary at 90 degree elbow angle ($\theta = 90$); therefore, the muscle forces have to balance the weight of the forearm, hand, and the hand-held weight. The forearm is actuated by three flexor muscle groups (biceps brachii (BIC), brachiradialis (BRD), brachialis (BRA)).

The dynamics of the forearm can be represented as:

$$\ddot{\theta} = \frac{1}{I}\left(\sum(F_i R_i) - wd\sin(\theta)\right) \tag{1}$$

where θ is the elbow angle (as shown in Fig. 1). The muscle forces, F_i, $i \in \{brd, bic, bra\}$, produce joint moments about the elbow with the moment arms being R_i. The moment of inertia, I, and the weight, w, represent the total of forearm/hand/load. I is calculated with respect to the elbow joint axis, and the weight is acting at the centre of mass of the system located at a distance d

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TABLE 1. LIST OF SIMULATION PARAMETERS AND THEIR NUMERICAL VALUES

Parameter	value	Source
I	1.052 kg.m^2	Experiment/anthropometry* [19]
d	0.279 m	Experiment/anthropometry* [19]
w	118.78 N	Experiment/anthropometry* [19]
R	various	Garner and Pandy [20]
$F_{0_{max}}$	various	Garner and Pandy [20] (modified)
k_f	0.016 s^{-1}	Xia and Frey Law [13]
k_r	0.0024 s^{-1}	Xia and Frey Law [13]
L_d	60 s^{-1}	Xia and Frey Law [13] (modified)
L_r	60 s^{-1}	Xia and Frey Law [13] (modified)
A	0.5	
B	0.5	

*For 95 kg male

from the elbow joint. The numerical values of all the parameters in this simulation are given in Table 1.

To calculate the muscle force from the neural excitation signal, a Hill-type muscle model has been used. In our muscle model, we have combined the formulation developed by Thelen [21] and the fatigue model of Xia and Frey Law [13], which is explained here briefly.

The muscle force is mainly modulated via the activation level of the muscle, a. Furthermore, the muscle force is also affected by its length and shortening velocity according to (2).

$$F = a f_l f_v F_{max} \tag{2}$$

In this relation, F is the muscle force, and the terms f_l and f_v adjust the muscle force based on its length and velocity, respectively. These relations are given in detail in [21] and are omitted here for brevity. Lastly, the maximum isometric muscle force, F_{max}, is used to define the force production capacity of the muscle.

We have assumed that the muscle activation level correlates with the number of active muscle fibres. In the human body, the CNS recruits the muscle fibres in a specific order, known as the size principle [22]. At low force requirements, smaller muscle fibres are recruited, and as the muscle force increases, larger fibres (that can generate more force) are activated. During

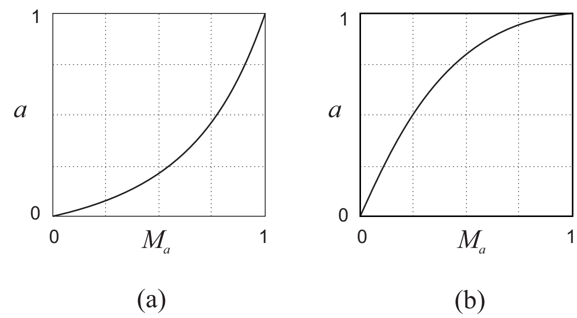

FIGURE 2. THE NON-LINEAR RELATIONS BETWEEN THE MUSCLE ACTIVATION LEVEL, a, AND THE NUMBER OF ACTIVE MUSCLE FIBRES, M_a. (a) FOR NORMAL RECRUITMENT ORDER. (b) FOR REVERSE RECRUITMENT ORDER (STIMULATION BY FES)

the application of electrical pulses via FES, the recruitment order is reversed, as the larger fibres have lower activation threshold. Therefore, for normal recruitment order, we have used the non-linear curve of (3) to account for the increased produced force.

$$a = A\, M_a^2 + B\, M_a \tag{3}$$

In equation (3), we have used the dimensionless variable M_a to represent the ratio of the number of active muscle fibres to the total number of muscle fibres (therefore it ranges from zero to one). The parameters A and B are used to define the non-linear relation between the number of active muscle fibres to the activation (force production) level of the muscle. Figure 2 shows the two curves for normal and reversed recruitment orders; the reverse order can be used to model the activation of muscle during the application of FES.

The number of active muscle fibres, M_a, changes based on the neural excitation signal. The three-compartment representation of [13] has been used here to model the fibre transfers between the active, rested, and fatigued states. Figure 3 shows the essence of this model.

The muscle fibres move from resting to active state according to the drive C. The fatigue process involves the transfer of a portion of the active fibres to the fatigued states. Similarly, a portion of the fatigued fibres move from the fatigued state to the resting state via recovery process. Therefore, the transfer dynamics can be summarized as:

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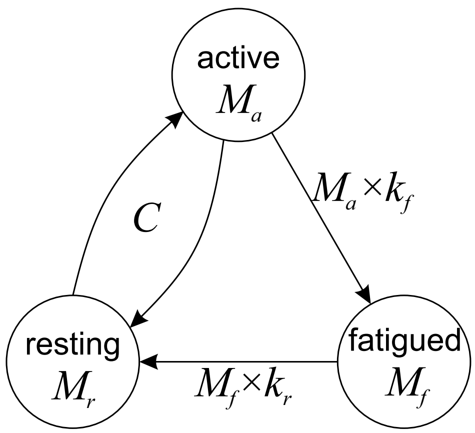

FIGURE 3. THE THREE STATES OF THE MUSCLE FIBRES ACCORDING TO THE THREE-COMPARTMENT FATIGUE MODEL

$$\frac{dM_a}{dt} = C - k_f M_a \tag{4}$$
$$\frac{dM_r}{dt} = -C + k_r M_f \tag{5}$$
$$\frac{dM_f}{dt} = k_f M_a - k_r M_f \tag{6}$$

where M_r, M_a and M_f are the resting, active, and fatigued muscle fibres, respectively. The transfer rates k_f and k_r represent the fatigue and recovery rates, and C is the drive expressed as:

$$C = \begin{cases} L_r(u - M_a) & u < M_a \\ L_d(u - M_a) & M_a \le u < M_a + M_r \\ L_d M_r & M_a + M_r \le u \end{cases} \tag{7}$$

In the definition of the drive C, the first case represents the relaxing process, in which the number of active fibres, M_a, is larger than the demand u. Therefore, C is negative, resulting in a transition of muscle fibres from active to resting state; the rate at which the relaxing transition occurs is denoted by L_r. The second case represents normal force development, when the demand u is more than the current activity M_a, and there is enough non-fatigued muscle fibres to generate force. In this case some fibres move from resting state to active state, with the development rate being L_d. Finally in the last case, there is not enough non-fatigued fibres to satisfy the demand; therefore, all the resting fibres will move to the active state to generate whatever force the muscle can. The numerical values of the fatigue parameters are also provided in Table 1. The numerical values for L_d and L_r are modified from the original values in [13], to match the delays observed in excitation/activation process [23].

The muscle force production capacity, F_{max} in (2), also reduces as the number of fatigued fibres is increased. Therefore, to account for this reduction, the maximum isometric muscle force at rest, $F_{0_{max}}$, can be scaled using the number of non-fatigued muscle fibres as:

$$F_{max} = F_{0_{max}}(1 - M_f) \tag{8}$$

Optimization Method

The usual practice to solve for the muscle forces in a multi-muscle system is to minimize a criterion—usually an index of effort. The results of the activation-based optimization process have been shown to match experimental electromyograms (EMG) (as far as the model limitations allow) [1,5]. It is usually argued that the squared or cubed activation levels represent muscular effort, and minimization of the objective function (9) will minimize the fatigue.

$$J = \sum_i a_i^p, \quad p = 2 \text{ or } 3, \quad i \in \{brd, bic, bra\} \tag{9}$$

It should be noted that the fatigue process (similar to energy expenditure) is, in fact, a time-dependent process. Thus, the static cost function of (9) may not properly reflect the dynamics involved in the fatigue process. Dynamic optimization [8] can therefore be used instead of static optimization to consider the history of signals by minimizing:

$$J = \sum_i \int_{t_0}^{t_f} a_i^p, \quad p = 2 \text{ or } 3, \quad i \in \{brd, bic, bra\} \tag{10}$$

As an alternative objective function, we have used the dynamical fatigue model presented in the previous section to represent the fatigue dynamics more accurately. Instead of minimizing the activations, we study the history-dependent objective function of:

$$J = \sum_i \int_{t_0}^{t_f} (M_{f_i})^p dt, \quad i \in \{brd, bic, bra\} \tag{11}$$

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where p is assumed to be $p = 3$.

To capture the effects of fatigue development, the model equations have to be integrated for the entire simulation time. In this case, the optimization goal is to find the control inputs (muscle excitation signals) for the entire simulation time. However, finding the control inputs at each time step is infeasible because of the large number of variables.

To reduce the number of optimization variables, we can approximate the signal using a parametrized curve, e.g. polynomial, Fourier series, spline/B-spline, or Bezier curves. Polynomials are in general very sensitive to the changes in the parameters, and are not suitable for characterizing our signals. B-spline (and Bezier as a general case of B-spline) show gentle behaviour in regard to changes in their parameters, but are more difficult to implement. Fourier series [4, 24] are easy to implement and computationally efficient. Thus, we have used Fourier series to parametrize the muscle excitation signals. The goal of the optimization is reduced to finding the Fourier coefficients (α_i and β_i in (12)) that characterize the optimal excitation signals.

$$u = \alpha_0 + \sum_{j=1}^{n} \alpha_j \sin(j\omega_0 t) + \beta_j \cos(j\omega_0 t) \tag{12}$$

To summarize the optimization problem, the cost function (11) is minimized subject to the following constraints:

$$0 \leq u_i \leq 1 \tag{13}$$

$$\ddot{\theta} = \frac{1}{I}\left(\sum(F_i R_i) - wd\sin(\theta)\right) = 0 \tag{14}$$

$$i \in \{brd, bic, bra\}$$

EXPERIMENTS

A 26 year old male subject performed four trials of stationary holding of a 10 kg weight, at 90-degree elbow flexion angle and 90-degree forearm supination; the subject continued the task until fully fatigued (could no longer hold the weight). Sufficient time (>10 minutes, as requested by the subject) was given for recovery between the trials. This exercise is aimed to activate the biceps the most, and we expected the most fatigue in the biceps. The subject also confirmed muscle fatigue in his biceps after the exercise. During the exercise, the EMGs of two muscles (biceps, brachioradialis) were recorded, and compared against the simulated muscle activities. The collection sampling rate was 4000 Hz, and the recorded signals were zero-biased, full wave rectified, and low-pass filtered. The processed EMG data are shown in Fig. 4.

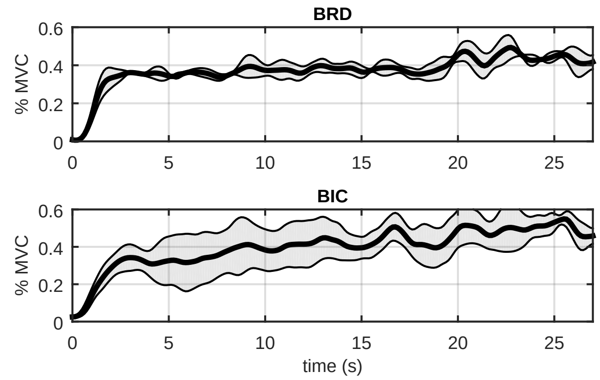

FIGURE 4. THE MEAN AND STANDARD DEVIATION OF THE RECORDED EMG SIGNALS FOR TWO MUSCLES

TABLE 2. COMPARISON OF THE INDICES FOR THE TWO SIMULATIONS

Index	simulation #1 Minimizing activation	simulation #2 Minimizing fatigue
$\sum_i \int_{t_0}^{t_f} ({M_f}_i)^3 dt$	0.2451 s	0.2305 s
$\sum_i \int_{t_0}^{t_f} (a_i)^3 dt$	11.202 s	16.045 s

RESULTS

We have run two sets of simulations in this study to show the difference between the minimization of time-dependent fatigue (11) and minimization of activations (10). Figure 5 (left column) shows the simulation result when the activation-based objective function of (10) is minimized. To obtain these results, the same dynamic optimization method has been used. The right column of Fig. 5 shows the simulation results when the objective function has been replaced with the time-dependent objective of (11). In both simulations, a fifth-order (n=5) Fourier series has been used.

As can be seen, the constraints are satisfied in both cases; however, the muscle activities are drastically different. Clearly, the co-activation obtained from the activation-based optimization better matches the experimental EMGs (Fig. 4), and this objective function seems to be a better index in the prediction of muscle activities. The minimization of the time-dependent muscle fatigue cannot be the strategy used by the CNS in the control of actions, even when the muscles are fatigued.

The calculation of the fatigue index ($\sum_i \int (M_f)^3 dt$) shows that the muscle fatigue is indeed lower (see Table 2) when muscle fatigue has been minimized, which confirms the successfulness of fatigue minimization strategy. In this case, the strongest muscle (biceps brachii) is loaded the most, and is helped (in shorter periods) by other muscles. It is also interesting to note that the fatigue minimization criterion results in higher activity levels.

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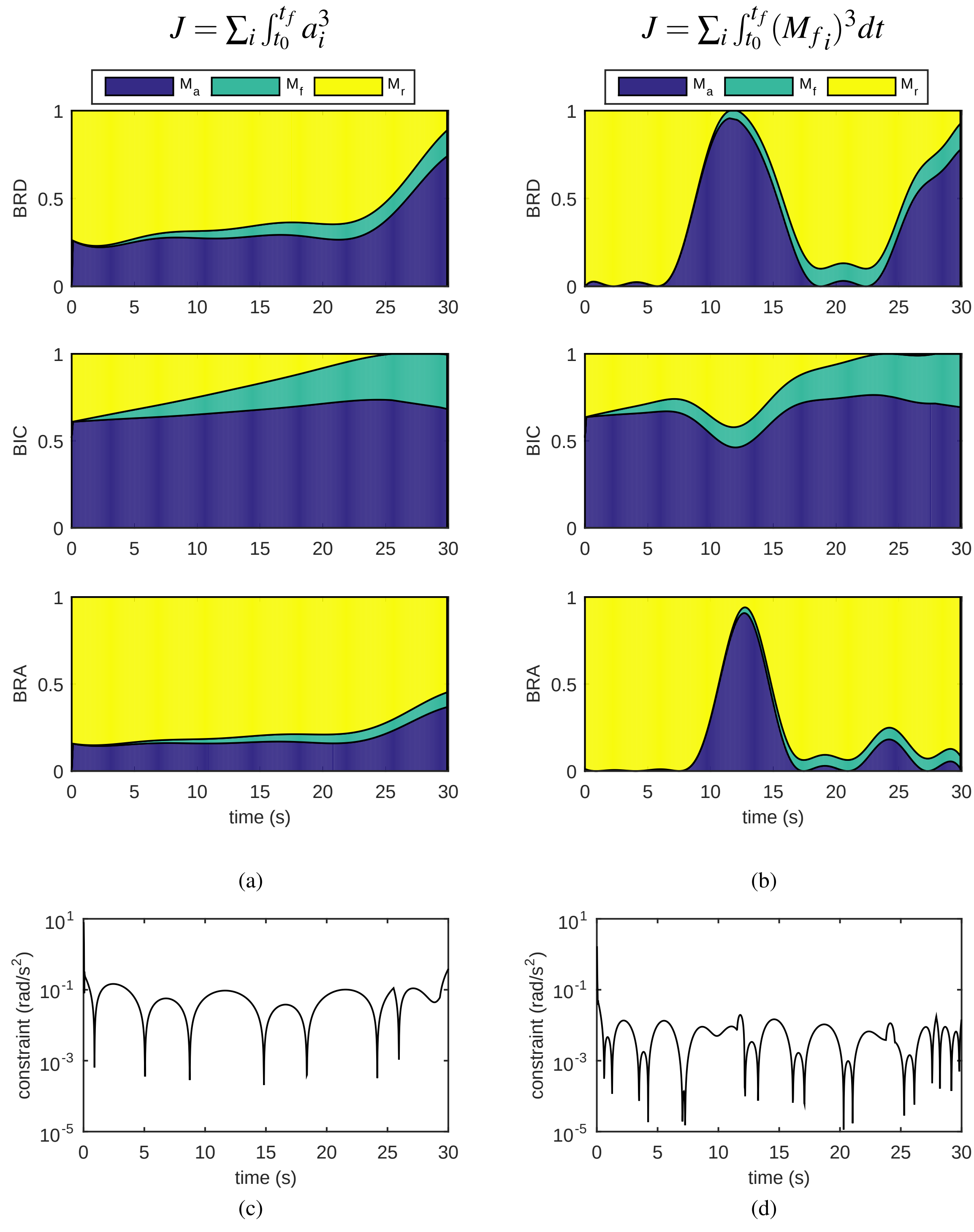

FIGURE 5. THE SIMULATION RESULTS FOR THE TWO MINIMIZATION OBJECTIVES. (a,b) THE ACTIVE, RESTING, AND FATIGUED MUSCLE FIBRES FOR THE THREE MUSCLES. LEFT: MINIMIZING ACTIVATIONS, RIGHT: MINIMIZING THE FATIGUE. (c,d) THE VALUE OF THE EQUALITY CONSTRAINT (14)

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DISCUSSION

We presented a new minimization criterion to find muscle activations in a musculoskeletal system. We used a dynamical fatigue model to study the effects of prolonged muscle activities, and found the muscle activity patterns that minimized muscle fatigue. The results of this research revealed that co-activation of muscles does not necessarily reduce muscle fatigue (as defined by our fatigue model) after a prolonged exercise. The optimization results showed that muscles experience less overall fatigue, if the strongest muscle is activated the most, with occasional help from the other muscles.

Although sharing the load between all the muscles results in lower individual muscle force, and consequently lower muscle activities, it manifests differently in muscle fatigue. Our results showed that the sum of the cubed activations does not result in the lowest fatigue, nor does the fatigue minimization method results in the lowest activation levels. In other words, the cubed activations does not properly represent muscle fatigue. An extended study is needed to explain the reason behind the intermittent muscle activation patterns, seen in the fatigue-based results in Fig. 5(b).

Additionally, the findings of this study show that the minimization of muscle fatigue is not the best practice in simulations where realistic muscle activation patterns are required. Nonetheless, such an approach can have its own important implications. As mentioned in the introduction section, the main motivation for this research has been the rapid occurrence of fatigue during the application of FES. According to our results, it is not necessary to co-activate muscles in order to delay the fatigue process; instead, the strongest muscle can be used as the primary mover, while other muscles can be used to help the primary muscle in short intervals.

This argument raises the question if such an activation pattern is beneficial to the patient. The application of FES in rehabilitation programs is intended to facilitate the neuroplasticity, by co-stimulating the sensory and motor areas of the brain. However, this *unnatural* pattern of muscle activations may conflict with the existing motor control rules, and may result in maladaptation, or delay the neuroplasticity. Further investigations on the implication of such activation patterns in the application of FES are necessary to make stronger arguments.

CONCLUSIONS

In this study we used a mathematical model of muscle fatigue to study the effects of prolonged exercise. Since FES is prone to muscle fatigue, we were motivated to find the neural excitation patterns that minimize muscle fatigue, using a dynamic optimization approach.

The dynamical fatigue model allowed us to include a time-dependent objective function instead of an instantaneous activation-based objective. Due to the dynamics of fatigue, it was concluded that the fatigue is minimized when the strongest muscle is activated the most.

To make a stronger argument, however, more experimental and simulation studies must be conducted. First of all, the validity of the fatigue model must be tested more rigorously than the original experiment in [13]. More complex fatigue models can also be used to see if the same results are obtained. More importantly, new experiments must be designed to test whether the recommendations made in this paper are useful in reducing fatigue after application of FES.

ACKNOWLEDGEMENTS

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