2012 American Control Conference
Fairmont Queen Elizabeth, Montréal, Canada
June 27-June 29, 2012

On Real-Time Optimal Control of a Series Hybrid Electric Vehicle with an Ultra-Capacitor

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Abstract— **To design a supervisory plan for a Hybrid Electric Vehicle (HEV), different methods are presented in the literature. In many of these controllers, there are a few parameters that must be tuned according to future driving conditions. To address this issue, a novel feedback controller is introduced in this paper that serves as the supervisory plan of a series HEV, and does not require the exact knowledge of the drive cycle. To find this controller, first a mathematical model of the hybrid drivetrain is developed, then Pontryagin's Minimum Principle is applied assuming that the drive cycle is known in advance. Based on the mechanism of the optimal control, a set of mathematical rules is extracted, and an optimal feedback controller is designed. It is also shown that a priori knowledge of the future drive cycle is not required; it is possible to tune the controller parameters, knowing only the cruise time and the available negative energy during braking.**

I. INTRODUCTION

It is now widely accepted that development of Hybrid Electric Vehicles (HEVs) is a promising short term solution to reduce the fuel consumption and vehicular air pollution. Although HEVs still rely on conventional internal combustion engines and fossil fuels, they act as a technological bridge, providing a transition between conventional cars and the newer technologies such as full electric and fuel-cell powered vehicles. HEVs and Plug-in HEVs (PHEVs) can have the following advantages compared to conventional vehicles: 1) Possibility of downsizing the engine, 2) Possibility of charging the electrical energy storage with an external source (in PHEVs), 3) Ability of capturing the kinetic energy during braking, 4) Increased engine efficiency due to extra degrees of freedom in the drivetrain.

The last two points indicate the need for an accurate plan (the so called *supervisory controller* in the literature) which determines the proper amount of power to be generated by either of the two onboard sources. Rule-based controllers are good candidates for such supervisory plans since they are easy to implement. However, because of the tuning problems and sub-optimal behaviors of these rule-based controllers, model-based controllers have gained considerable attention in recent years [1].

Among model-based controllers, stochastic dynamic programming (SDP) [2] and model predictive control (MPC) [3], [4] are causal controllers, capable of providing near-optimal solutions. However, they are computationally expensive, and the results may be inferior to global optimal solutions [4]. Pontryagin's Minimum Principle (PMP) is a useful approach for HEV applications, which can result in the global optimal solution under some conditions [5]-[8]. The Equivalence Consumption Minimization Strategy (ECMS) is another promising method [5], [9], [10]. In ECMS, the equivalence factor is the parameter that should be tuned in order to get optimal behaviors. The tight correlation between the costate in the PMP approach and the equivalence factor in ECMS is shown in [5] and [10]. Tuning of those parameters is based on the information about the drive cycle in the future.

In this paper, a real-time supervisory controller is presented. This controller is based on the findings of the PMP method, hence gives the optimal solution. For its tuning, a method is presented that does not require a priori knowledge of the whole drive cycle; instead, it only requires the time until the next stop, and the amount of regenerative energy available during braking.

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II. Control-Oriented Model

To obtain a model-based controller, a mathematical representation of the hybrid drivetrain is required. The control-oriented model should capture important characteristics of the drivetrain, and remain simple enough so that the computation time stays below realtime requirements.

In this work, a series HEV is studied (fig 1). An ultra-capacitor (UC) is used as the energy storage. UCs are ideal for HEV application because they have higher efficiency and power-density, and much longer lifetime compared to NiMH and Li-ion batteries. In contrast, their relatively low energy-density and internal energy dissipation make them inappropriate for EV and PHEV applications [11].

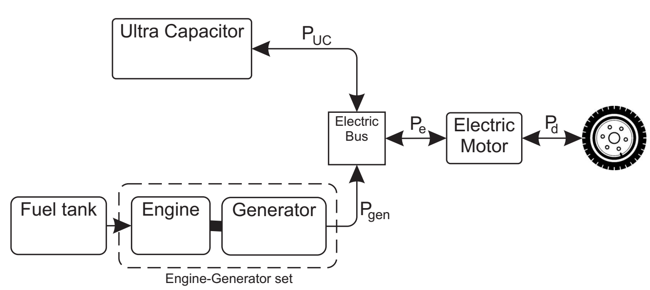

Fig. 1. Schematic of a series HEV

TABLE I
PARAMETERS USED IN THE SIMULATIONS

Parameter	Value	Parameter	Value
m_v	1600 kg	$P_{gen_{max}}$	50kW
α	7e-5 g/s/W	C	200F
β	0.15g	R	0.1Ω
f_{rr}	0.01	$V_{C_{max}}$	200V
ρ	1.15 kg/m^3	x_{max}	$\sqrt{0.7}V_{C_{max}}$
A	2.31m2	x_{min}	$\sqrt{0.5}V_{C_{max}}$
C_d	0.32	x_{ref}	$\sqrt{0.6}V_{C_{max}}$
$P_{UC_{max}}$	60kW	η_m	0.96
$P_{UC_{min}}$	-60kW	G	1000

In the first part of the study, the backward quasi-static model of the drivetrain is used to calculate the necessary power based on the vehicle velocity. This power is used as the input to the optimal control problem.

To calculate the power demand, the longitudinal vehicle dynamics is modeled as:

$$m_v a_x = f_T - (f_D + f_R + m_v g \sin(\gamma)) \tag{1}$$

And (2a) is used to find the power demand.

$$P_d = v f_T = v(m_v a_x + f_D + f_R + m_v g \sin(\gamma)) \tag{2a}$$

$$f_D = \frac{1}{2}\rho v^2 A C_D \tag{2b}$$

$$f_R = m_v g \cos(\gamma) f_{rr} \tag{2c}$$

In the above equations, m_v is the mass of the vehicle; v and a_x are the longitudinal velocity and acceleration respectively; f_T is the traction force, resulting from the torque on the wheels; f_D is the aerodynamic drag force; f_R is the equivalent rolling resistance of all wheels, and the term $m_v g \sin(\gamma)$ is the resistive force due to the slope of the road. ρ, A, and C_D are air mass density, vehicle frontal area, and drag coefficient respectively. Numerical values for all the parameters used in this study are presented in table I.

To model the hybrid drivetrain, quasi-static model of each component is used [8]. In the following, the model of each component is presented.

1) Ultra-Capacitor: Ultra-capacitors can be modeled using RC circuits (fig 2(a)), and the number of the RC branches can determine the accuracy of the model [12]. Since increasing the number of the branches increases the number of the states of the system, only a simple RC circuit is used in the control-oriented model (fig 2(b)). This model can still capture enough detail about the behavior of the UC. The relation between the voltage of the UC and the current passing through it is according to:

$$\dot{V}_C = -\frac{i}{C} \tag{3}$$

And the output power of the UC, P_{UC}, is found using (4).

$$P_{UC} = iV_C - Ri^2 \tag{4}$$

In these relations, the values C and R are the equivalent capacitance and resistance of the UC respectively. V_C is the capacitance voltage and i is the current passing through UC.

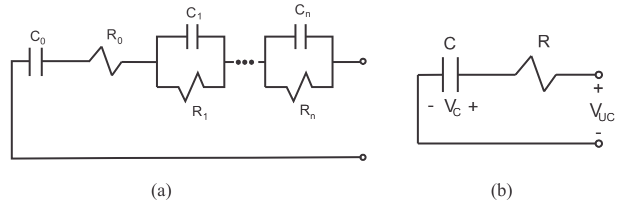

Fig. 2. Ultra-capacitor circuit model

The negative sign in (3) states that a positive current (power) discharges the UC, and a negative current charges it.

In this HEV model, the only state is the voltage of the capacitance, V_C. The control parameter is chosen to be the current of the UC, thus:

$$\dot{x} = -\frac{u}{C} \tag{5}$$

State of charge (state of energy) is defined as the ratio of the current charge (energy) of the storage to its maximum value, i.e.:

$$SoC = \frac{\text{stored charge}}{\text{maximum charge}} = \frac{V_C}{V_{C_{max}}} \tag{6a}$$

$$SoE = \frac{\text{stored energy}}{\text{maximum energy}} = \frac{{V_C}^2}{{V_{C_{max}}}^2} = SoC^2 \tag{6b}$$

2) Engine-Generator: As the engine is rigidly coupled to the generator, and they are not mechanically connected to the drive-line, their speed can be chosen arbitrarily. This is one of the major advantages of the series architecture, because the engine speed can be chosen so that the engine can work with maximum efficiency for a given load, hence reducing the fuel consumption. This means that for a given load, the operating point and the fuel consumption is determined. The plot of the the steady-state fuel consumption rate versus the generator output power is shown in fig 3. This plot is found by simulating a mean-value engine model, [13], coupled to a permanent magnet DC generator. It is clear that the relation is almost linear; therefore the fuel consumption can be approximated as:

$$\dot{m} = \alpha P_{gen} + \beta \tag{7}$$

with α and β being constants.

3) Electric Motor: The electric motor can be modeled as a transducer that converts the electrical power to mechanical

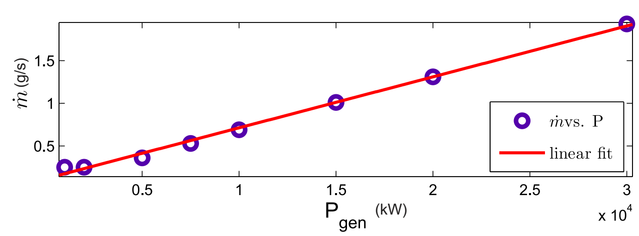

Fig. 3. Fuel consumption rate versus generator power

power and vice versa. Losses of the drive-line and the motor can be modeled as a single efficiency using (8).

$$P_e = P_d \; \eta_m^{-sign(P_d)} \tag{8}$$

In this equation η_m is the total efficiency of the drive-line from the electric motor to the wheels, and it was seen that a constant value can be accurate enough for control purposes.

4) Electrical Bus: When neglecting the losses, the balance of the energy in the electric bus can be written as:

$$P_{gen} + P_{UC} = P_e \tag{9}$$

In this relation, the positive values indicate that the power is flowing from the drivetrain toward the wheels and the negative sign shows that the power is reversed. It is obvious that the generator power cannot be negative.

III. Optimal Control Problem

To design a supervisory plan for HEVs, usually a cost function of the form (10) is considered.

$$J = h(SoC_f) + \int_0^{t_f} \dot{m}\, dt \tag{10}$$

The first term, $h(SoC_f)$ is used to penalize the variation of the final sate of charge from its reference value, and can be defined as:

$$h(SoC_f) = \frac{1}{2}G'(SoC_{ref} - SoC_f)^2 = \frac{1}{2}G(x_{ref} - x_f)^2 \tag{11}$$

$$\tag{12}$$

with G being a weighting factor. By combining (4), (5), (7), (8), (9), and (11) with (10), the cost function can be written as:

$$J = \frac{1}{2}G(x_{ref} - x_f)^2 + \int_0^{t_f} \left[\alpha(P_e - xu + Ru^2) + \beta\right] dt \tag{13}$$

This cost function has to be minimized, considering the physical constraints below:

$$x_{min} < x < x_{max} \tag{14a}$$

$$P_{UC_{min}} < P_{UC} < P_{UC_{max}} \tag{14b}$$

$$0 < P_{gen} < P_{gen_{max}} \tag{14c}$$

If (14b) and (14c) are considered together, and by using (9) and (4), it is possible to write them as a single constraint on the control as (16).

$$P_{min} = \max\{P_{UC_{min}}, P_e - P_{gen_{max}}\} \tag{15a}$$

$$P_{max} = \min\{P_{UC_{max}}, P_e, \frac{x^2}{4R}\} \tag{15b}$$

$$u_{min}(x,t) = \frac{x}{2R} - \frac{1}{2R}\sqrt{x^2 - 4RP_{min}} \tag{16a}$$

$$u_{max}(x,t) = \frac{x}{2R} - \frac{1}{2R}\sqrt{x^2 - 4RP_{max}} \tag{16b}$$

$$u_{min} < u < u_{max} \tag{16c}$$

Therefore the optimal control problem is defined as follows: *Find the optimal control, u, such that the cost function* (13) *is minimized while the constraints* (17) *are satisfied.*

$$\dot{x} = -\frac{u}{C} \quad x(0) = x_{ref} \tag{17a}$$

$$x \in \mathcal{X}\ ,\ \mathcal{X} = [x_{min}, x_{max}] \tag{17b}$$

$$u \in \mathcal{U}\ ,\ \mathcal{U} = [u_{min}, u_{max}] \tag{17c}$$

With the problem fully defined, Pontryagin's Minimum Principle can be used. It starts with the definition of the Hamiltonian:

$$H = \left[\alpha(P_e - xu + Ru^2) + \beta\right] + \lambda(-\frac{u}{C}) \tag{18}$$

where λ is the Lagrange multiplier or the *costate*. The minimization problem is therefore converted to instantaneous minimization of the Hamiltonian as:

$$u^* = \arg\min_{u \in \mathcal{U}} \{H\} \tag{19}$$

with the costate dynamics defined by (20).

$$\dot{\lambda} = -\frac{\partial H}{\partial x} = \alpha u \quad \lambda(t_f) = G_1(x_f - x_{ref}) \tag{20}$$

To get the solution, a two-point-boundary-value problem should be solved (the initial state and the final costate values are known). The shooting method can be used to easily solve the problem. In the shooting method, first the initial value of the costate is guessed. At each step, the value of the control u in the range $[u_{min}, u_{max}]$ that minimizes the Hamiltonian, (18), is chosen as the optimal value. Using this value, the state and costate equations are integrated to the next time step, and this procedure repeats until the final time of the mission is reached. If the final value of the costate does not match the criteria in (20), the whole process is repeated with a new guess for the initial value of the costate, until the criteria is satisfied.

With this method, the constraint on the control (17c) is explicitly satisfied. In order to keep the state (ultra capacitor voltage) in its desired range, a heuristic method is used. Whenever the voltage reaches its upper limit, the UC cannot be charged any longer; this means that the current cannot be negative, which implies that u_{min} must be zero. In the same way, u_{max} is set to zero when the voltage reaches the lower limit. This method, although being heuristic, can be proven mathematically to be a successful way to satisfy state inequality constraints. The detailed proof of this method can be found in [14, § 2.5].

IV. Optimal Feedback Control

At every time step, the value of the control that minimizes the Hamiltonian is chosen as the optimal value. Since at each time step, the state, the costate, and the power (x, λ and P_e in (18)) have constant values, the Hamiltonian take a quadratic form in terms of the control, u. It is worth noting that by using shooting method, the global optimality of the solution is guaranteed. That is because there is only one candidate of the solution, and if the costate satisfies its final value criteria, the solution is unique, thus globally optimal.

The quadratic form of the Hamiltonian also implies that the minimum of H happens either at a boundary value of u (namely u_{min} or u_{max}), or when $\frac{\partial H}{\partial u}$ is zero. When the derivative is zero, we have:

$$\frac{\partial H}{\partial u} = 0 \Rightarrow \bar{u} = \frac{x}{2R} + \frac{\lambda}{2R\alpha C} \tag{21}$$

$$\frac{d\bar{u}}{dt} = \frac{1}{2R}\frac{dx}{dt} + \frac{1}{2R\alpha C}\frac{d\lambda}{dt} \tag{22a}$$

$$= \frac{1}{2R}\left(-\frac{u}{C}\right) + \frac{1}{2R\alpha C}(\alpha u) \tag{22b}$$

$$= 0 \tag{22c}$$

The above relation means that the value of $\bar{u}$ is constant in the whole mission. In fact, this value is the governing parameter in this problem, and can be found by (21), using only initial values of the state and the costate. Now the mechanism of the optimal control can be concluded as follow:

1) The optimal controller tries to use the ultra capacitor as much as possible. This happens when $u^* = u_{max}$, and the whole power, P_e, is provided by the UC or the whole power (during braking) is absorbed by UC, fig 4 (a).
2) The optimal controller limits the power delivered by the UC (i.e., when $u^* = \bar{u}$), so that the charge sustenance is guaranteed, fig 4 (b).
3) The optimal controller gives higher priority to drivability (following the driver commend), i.e. when there is a high power request, the UC assists the generator disregarding the limit mentioned in point 2, fig 4 (c).

With these simple rules, it is possible to define a simple yet optimal feedback controller as:

$$u^* = \max\{u_{min}(x,t), \min\{u_{max}(x,t), \bar{u}\}\} \tag{23}$$

with u_{min}, u_{max} and $\bar{u}$ defined in (16a), (16b) and (21) respectively. The inputs to the controller are the state (as feedback) and P_d, and it determines the optimal current of the battery.

As was mentioned earlier, the most important parameter in this controller is the initial value of the costate (which in turn defines the value of the $\bar{u}$). In the next section, a simple method is presented to find the proper value of the costate.

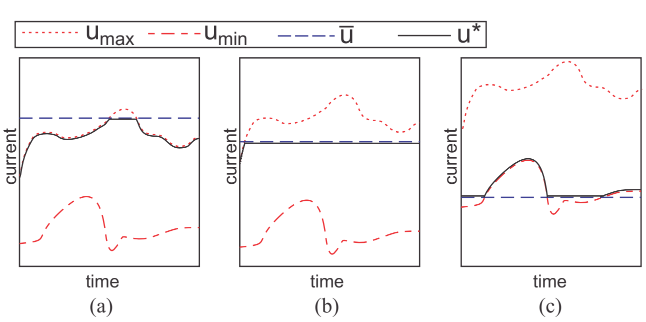

Fig. 4. The mechanism of the optimal supervisory plan

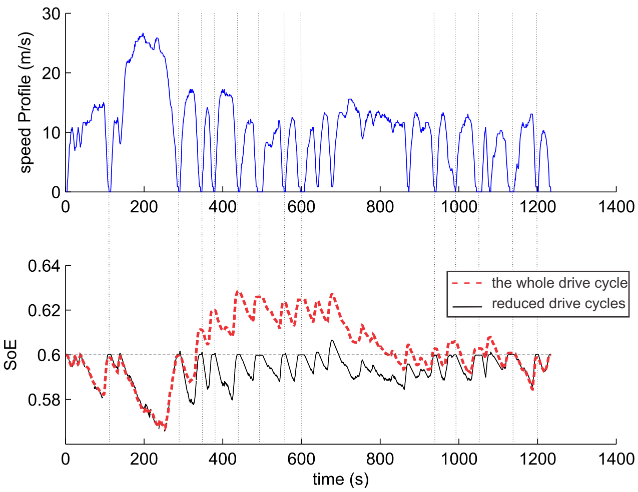

Fig. 5. State trajectory when considering the whole drive cycle, and when considering reduced drive cycles

TABLE II

FUEL CONSUMPTION FOR THE TWO METHODS

	UDDS	HUDDS	EUCD	NYCC
the whole drive cycle (gr/cycle)	383.8	296.6	456.5	92.24
reduced drive cycle (gr/cycle)	384.0	297.3	457.9	92.48

V. COSTATE ESTIMATION METHOD

In order to have the optimal behavior, it is necessary to have the future driving condition. Without such information, only sub-optimal behavior is achievable [1]. In this study, it was observed that it is not necessary to have the whole drive cycle considered. Instead, if only the driving condition until the next stop is known, it is possible to obtain a solution which is just slightly inferior to the solution found considering the whole drive cycle. An example is presented in fig (5) for the UDDS drive cycle. A comparison of the resultant fuel consumptions for various drive cycles is also presented in table II. To obtain these results, the PMP was solved once for the whole drive cycle, and once for every *reduced* drive cycle (from one stop to the next) when the final state was required to be x_{ref}.

The initial value of the costate (in brief, *costate* hereafter) is the only parameter that should be tuned for these reduced drive cycles. Since the optimal control mechanism is independent of the driving condition, it is only charge sustenance that should be considered in tuning the costate.

The mechanism mentioned earlier follows one important concept: It tries to capture as much negative energy as possible; that elevates the SoE. In order to discharge the UC to its initial charge level, the controller decides that a *certain amount* of power is provided by the UC during acceleration and cruising. This certain amount is directly related to $\bar{u}$ through:

$$P_{UC} = x\bar{u} - R\bar{u}^2 \tag{24}$$

The objective is now estimation of the costate for each of

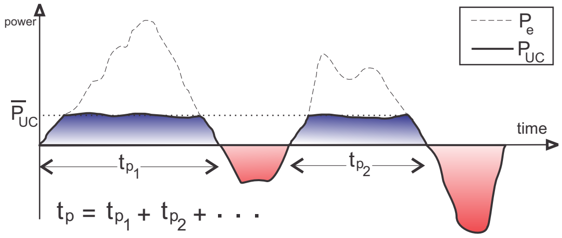

Fig. 6. A typical plot of P_{UC}

the reduced drive cycles to satisfy charge sustenance. A typical plot of electric power demand, P_e, and the corresponding UC power is shown in fig (6). To have SoE at the same level at the beginning and end of the mission, the total change in the ultra-capacitor energy (the integral of the shaded area) must be zero, i.e.:

$$E_n + E_p = 0 \tag{25}$$

E_n represents the total negative energy absorbed by the UC, and E_p is the total energy delivered to drivetrain by the UC. It is reasonable to assume that the positive power is always equal to $\bar{P}_{UC}$, which gives:

$$E_p = \int_{t_p} P_{UC} dt \simeq t_p \bar{P}_{UC} \tag{26}$$

where $\bar{P}_{UC}$ is:

$$\bar{P}_{UC} = (x_0 \bar{u} - R\bar{u}^2) \tag{27}$$

In (26), t_p is the time when power demand is positive. Although the voltage, x, changes with time, assuming a constant value for it (namely x_0) does not have a huge impact on the accuracy of the calculations. That is due to the fact that the change of x is limited to $\pm\sqrt{10}\%$ of its maximum value. Combining (26) and (27) with (25) gives:

$$E_n + t_p(x_0\bar{u} - R\bar{u}^2) = 0 \tag{28}$$

Thus the value of $\bar{u}$ that satisfies charge sustenance requirement can be found according to (29).

$$\bar{u} = \frac{x_0 - \sqrt{{x_0}^2 + 4R\frac{E_n}{t_p}}}{2R} \tag{29}$$

This equation relates the optimal value of $\bar{u}$ to only two parameters of the drive cycle: the total negative energy available and the time when positive power is required. During the simulations, it was observed that the optimal value of $\bar{u}$ is independent of the order of events; for example, it is not necessary to know when the driver is going to push the brake pedal; it is only important to know how much kinetic energy is going to be transferred to electrical energy before the next stop. This can be justified by (29), which is only a function of total energy and time.

In relation (25), it is assumed that the final SoE should come back to its initial level. If (due to any kind of error) the initial SoE has a value different from the desired SoE_{ref} (corresponding to x_{ref}), then the feedback controller tries to bring it back to the initial value not SoE_{ref}. To compensate this error, the difference in the UC energy should be considered as:

$$E_n + E_p = E_{res} \tag{30}$$

with E_{res} being

$$E_{res} = \frac{1}{2}C({x_0}^2 - {x_{ref}}^2) \tag{31}$$

Therefore, $\bar{u}$ can be more robustly approximated using (32).

$$\bar{u} = \frac{x_0 - \sqrt{{x_0}^2 + \frac{4R}{t_p}(E_n - E_{res})}}{2R} \tag{32}$$

By substituting (21) in (32), a relation for the costate can be derived:

$$\frac{x_0}{2R} + \frac{\lambda_0}{2R\alpha C} = \frac{x_0 - \sqrt{{x_0}^2 + \frac{4R}{t_p}(E_n - E_{res})}}{2R}$$

$$\lambda_0 = -\alpha C\sqrt{{x_0}^2 + \frac{4R}{t_p}(E_n - E_{res})} \tag{33}$$

Using this value and integration of (20), the optimal costate can be found at each time.

It was mentioned in section I that ECMS can be optimized using the PMP approach. In these cases, the equivalence factor is tightly related to the costate. Therefore it is possible to find the optimal value of the equivalence factor at each instant using the costate trajectory found by the method presented in this paper.

Although the costate estimation method still requires some information about the future driving conditions, it is a less demanding problem than finding the exact speed profile. It is possible to estimate the cruise time using ITS and GPS systems; the available negative energy is related to the kinetic energy of the vehicle during braking, which can also be estimated using the longitudinal vehicle dynamics and such devices [15].

VI. Results

In this study, simulations are conducted in the Matlab environment. The optimal control problem is solved using PMP for the UDDS drive cycle, and control and state trajectories are shown in fig 7. Note that the optimal control value is the same as u_{max} but is limited to a constant value ($\bar{u}$).

For 68 reduced drive cycles (which was extracted from 11 standard drive cycles) the optimal costate is found, and the correlation between the costate and the drive cycle parameters, (33), is presented in fig 8. In this plot, the best linear approximation does not exactly match with relation (33). This is mostly because of the simplifications used to find (33). It is possible to increase the accuracy of the estimation, by finding a more accurate value of the integral of the shaded area in fig 6. However, it was observed that it demands more information about the drive cycle, but does not give much more accuracy in return.

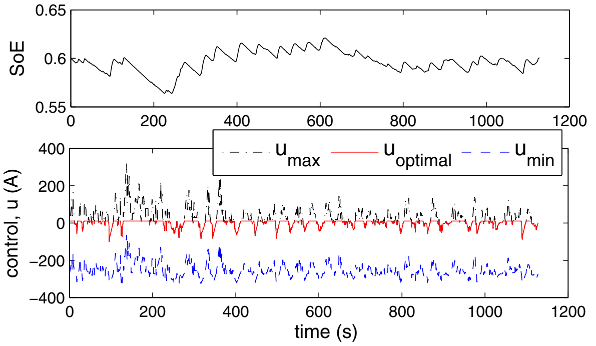

Fig. 7. The optimal SoE and control trajectories for UDDS drive cycle found using PMP solution

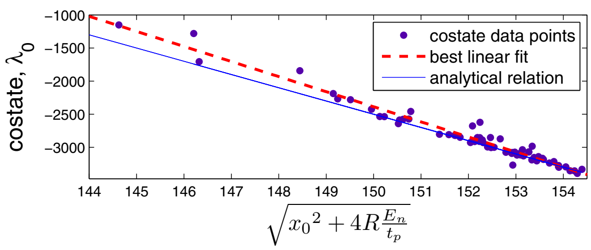

Fig. 8. Linear fit of the costate data

In fig 9, the control and state trajectories obtained by three different methods are shown for the first 350s of the UDDS drive cycle. The methods are the solution of PMP for the whole drive cycle, the solution of PMP for successive reduced drive cycles, and the solution of the feedback controller with estimated costates. Although many simplifications are made to estimate the costate, the results are very close to the ones found using PMP. The fuel consumption for different standard drive cycles are presented in table III. It should be noted that the difference in fuel consumption for the second and third methods are mostly due to variations of SoE rather than the optimality of the method. In cases with more difference in fuel consumption, the final SoE is higher than the reference value, which causes an increase in the fuel consumption.

VII. Conclusion

In this work, the problem of minimizing fuel consumption in hybrid electric vehicles was studied. To achieve the global optimal solution for HEVs, future driving conditions have to be known in advance. The controller presented in this paper gives near-optimal results for a series HEV with an ultra-

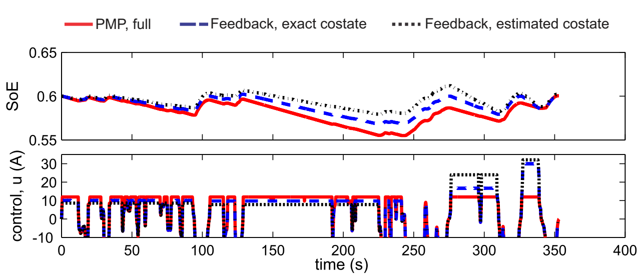

Fig. 9. Three different solutions for the first 350s of UDDS

TABLE III

Comparison of normalized fuel consumption for the different solutions

	UDDS	NYCC	EUC	HUDDS	HWFET
PMP solution, the whole drive cycle	1	1	1	1	1
PMP solution, reduced drive cycles	1.0004	1.002	1.003	1.002	1.00
Feedback controller, reduced drive cycles	1.001	1.016	1.014	1.011	1.031

capacitor. Furthermore, the amount of information needed is reduced from the whole drive cycle to only two parameters of the shorter stop-to-stop cycles: the cruise time, and the available regenerative energy in braking.

As a future work, this controller can be coupled to an adaptive cruise controller, and the required information can be extracted from GPS and ITS devices. It is also a part of the study to conduct a driver-in-the-loop simulation to study the controller performance with stochastic driver inputs.

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