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A motor control framework for the fast control of a 3D musculoskeletal arm motion using muscle synergy

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ABSTRACT — *This paper presents a general framework for the fast and efficient feedback control of musculoskeletal systems based on muscle synergies. In the proposed framework, a feedback control logic defines the reference trajectories in the task-space. In the lower level of the control hierarchy, the reference trajectory is translated to muscle activations via muscle synergies. The simulation results show that the proposed framework can control the the motion of a three degree of freedom and three dimensional musculoskeletal system quickly with near-optimal muscle activations.*

1 Introduction

Because of the larger number of muscles than the degrees of freedom in the human body, multiple solutions for muscle activations exist that produce a certain motion. A common practice to pick one solution out of the many possible solutions is to consider an extra criterion. The models that minimize an exertion index has been widely used in the literature [1]. The exertion indices such as muscle force [2, 3, 4], muscle activations [5, 6, 7], and physiological energy consumption [8, 7, 9] have all been used to estimate individual muscle activities.

There are two issues with the aforementioned models. One is, optimization is in general a very slow process, rendering it inapplicable for real-time control. The second issue is in the nature of the solution. Most of the optimization-based methods in the literature deal with inverse-dynamics simulations, where the motion is known. The few models that deal with forward-dynamics simulations only use optimization to find signals to drive the musculoskeletal system in a feed-forward manner [8, 10, 11, 12].

Feedback controllers for musculoskeletal systems are more scarce. [13, 14] have developed a hierarchical structure to control musculoskeletal systems, in which a simplified high-level model is used for the construction of optimal laws. In [15], task-related variables (i.e. center of mass position) have been used for balance control. In [16] acceptable controller performance is achieved, using well-tuned Proportional-Integral-Derivative (PID) feedback controllers to control the muscle activities.

Muscle synergy theory is a widely studied subject in motor control research [17]. According to this theory, the nervous system activates the muscles by combining a few number of bundles of activations (known as the synergies). This theory has been proposed as a way to simplify computations, by reducing the number of signals that the nervous system has to control [18].

Muscle synergy has mostly been studied in an inverse manner (i.e. finding the synergies by looking at the measured muscle activities). A few examples include studies on healthy humans [19], spinal cord injury [20] and cerebral palsy subjects [21], frogs [18, 22], and cats [23, 24]. However, few researchers have tried using synergies to produce motion in forward manner (use it to produce motion, instead of measuring it). The exceptions include [15, 12, 25], where simple tasks are studied.

In this paper, we propose a general framework for the feedback control of musculoskeletal systems using muscle synergies. Similar to the previous models [26, 14], our framework has a hierarchical structure to separate the control of the task-dependent variables from the complexities of the musculoskeletal system. We show how this motor control framework can be applied to a three-dimensional musculoskeletal arm model for general motion control in 3D space.

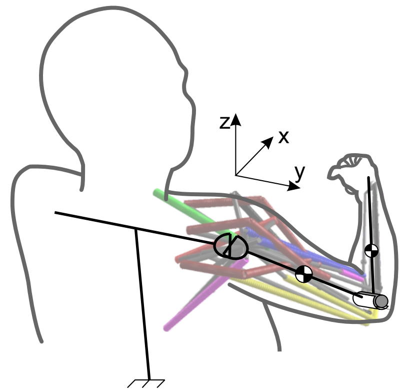


Fig. 1: The schematic of the 3-DoF arm model

2 Methods

2.1 Musculoskeletal model

We have used a three-dimensional (3D) musculoskeletal arm model to showcase our motor control framework. The model details are given in [3, 4]. Briefly, the model consists of the following segments: the torso, the upper arm, and the forearm/hand (see Fig. 1). The torso is assumed to be fixed, to which the upper arm is connected via a 2-DoF universal joint (allows for raising the arm in various planes of elevation). The elbow joint is modelled as a revolute joint allowing forearm flexion/extension, and the wrist is assumed to be rigid. Therefore, this 3D arm model has three degrees of freedom (DoF), which allows for reaching to any 3D point in the task space.

This model is actuated by 15 muscles (listed in [5]), which are modelled using the Hill-type formulation [27]. For simplicity, only the contractile element of the muscle model is implemented. The input to the model is the set of muscle activations, u, which result in muscle forces that generate motion. Forward dynamics is the predictive simulation method used in this work.

2.2 Motor control framework

The goal of the motor control framework is to drive the hand to any arbitrary target in the 3D task-space. Since this model does not have any redundant degrees of freedom, all of its degrees of freedom are controlled (in other words, the uncontrolled manifold [28] is empty).

The schematic of the proposed motor control framework is shown in Fig. 2. This framework has a hierarchical structure. The high-level controller is a feedback controller in the task-space. This task-space controller only deals with the task-space variables (3D hand position), and disregards all the complexities of the musculoskeletal system. It can be any feedback controller type, such as an error-driven (e.g. PID), optimal (e.g. model predictive controller, or linear quadratic regulator), or robust/adaptive controller. For the sake of simplicity, we have chosen a simple PID controller for this research. This controller compares the current location of the hand with the target, and defines the 3D acceleration required to reach to the target.

The human body motions are produced by muscle contractions. Thus, the reference acceleration specified by the high-level controller needs to be translated to muscle activations. Muscle synergies have been used to make this acceleration-to-activation mapping. It was previously suggested that task-specific synergies improve control performance in a musculoskeletal system [25]. Therefore, we have identified task-specific synergies to be used in this motor control framework.

Notes on task-specificity: By task-specific synergies we mean that the synergies employed during a 3D reaching task are probably different from the ones during elbow flexion, because the task-spaces controlled by the nervous system are different. The task-space in the former case is the (x, y, z) position of the hand, whereas the task-space in the latter is a 1D space, only including elbow joint angle—the hand position is irrelevant (not actively controlled) when the task is solely to flex the elbow. The uncontrolled manifold theory [28] can help distinguish between the different tasks by identifying the actively controlled variable.

2.2.1 Identification of synergies–an off-line process

Muscle synergies, if they exist, are probably formed either through evolution or practice. They must be structured in such a way that results in the best performance in doing a task, whether it be reducing the effort, or increasing the robustness. We believe that in the case of a familiar and well-trained action (e.g. reaching to a point in space), the synergies are structured to maximize efficiency for the intended task. Therefore, we have based our method on physiological effort minimization.

To obtain and store the synergies, a large number of optimization problems are solved to mimic the evolution (or training) process. At a given posture, the best combination of muscle activities that produce a certain task-space acceleration are found through an optimization process. The optimization algorithm finds the vector of muscle activations ($[\mathbf{u}]_{m\times 1}$, m = number of muscles) that minimize the following objective function:

$$[\mathbf{u}]_{m\times 1} = \arg\min \left\{ J = w_1 |\mathbf{a}_{des} - \mathbf{a}|^2 + w_2 \mathbf{u}^T \mathbf{u} \right\} \tag{1}$$

Here, J is the objective function that we are minimizing. $\mathbf{a}_{des}$ and $\mathbf{a}$ are the desired and actual hand acceleration vectors, respectively. w_1 and w_2 are weighting factors, balancing the importance of the tracking error and muscular effort terms. The optimization is constrained to the inequality constraint:

$$0 \leq u_i \leq 1 \quad , i \in \{1,..,m\} \tag{2}$$

The same process is repeated for n different acceleration vectors in various directions (see Fig. 3). The solutions of these optimization problems are then gathered in a matrix $\mathbf{A}$ as:

$$\mathbf{A}_{m\times n} = [\mathbf{u_1}, \mathbf{u_2}, \ldots, \mathbf{u_n}] \tag{3}$$

According to muscle synergy theory, the data matrix $\mathbf{A}$ is produced by a linear combination of a small number of synergies (or modules). Non-negative matrix factorization (NNMF, originally proposed in [29]) has extensively been used to extract the synergies from such a data matrix. NNMF finds the synergy matrix $\mathbf{S}$ and the coefficient matrix $\mathbf{C}$ with non-negative elements, such that their product best approximates the original data matrix $\mathbf{A}$:

$$\mathbf{A}_{m\times n} \simeq \mathbf{S}_{m\times k}\mathbf{C}_{k\times n} \tag{4}$$

or

$$\text{minimize: } e = \text{norm}(\mathbf{A}_{m\times n} - \mathbf{S}_{m\times k}\mathbf{C}_{k\times n}) \tag{5}$$

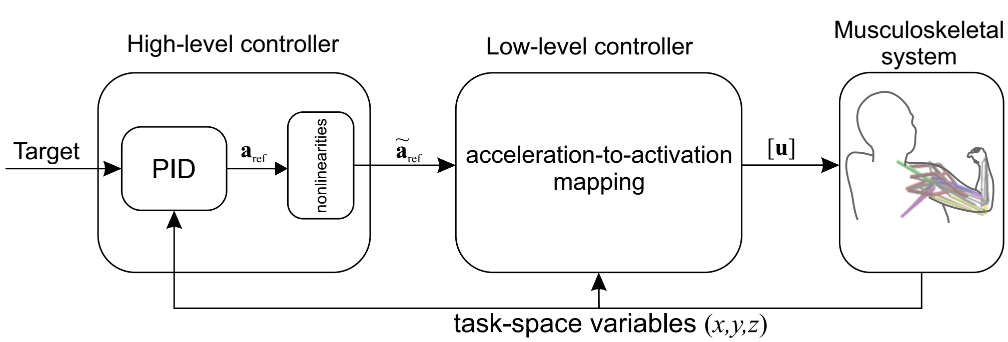


Fig. 2: The hierarchical structure of the proposed synergy-based motor control framework

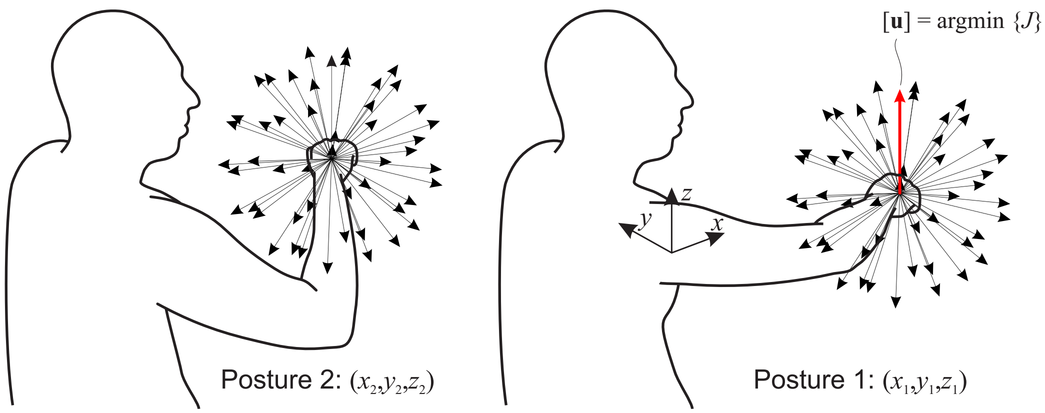


Fig. 3: A large number of optimization problems are solved off-line to obtain the synergies. In a given posture, multiple optimizations are solved to find the optimal muscle activations that produce different task-space accelerations. Same process is repeated for multiple postures, defined by hand position.

In this formulation, each column of the synergy matrix represents a synergy; $s_{i,j}$ contains the share of muscle i in synergy j. The elements of a column of the coefficient matrix contain the intensity of the corresponding synergies in one trial.

Applying the NNMF to the data matrix A gives a set of synergies at the current posture. Since the model has three DoF, specifying the hand position uniquely identifies the posture. If the entire process is repeated for all hand positions in the task space, a set of posture-dependent synergies can be defined (i.e. $S = S(x, y, z)$ where x, y, and z specify the position of the hand).

Notes on posture-dependent synergies: The available computational power limits us on the number of postures we can solve in a reasonable amount of time. Essentially, the synergies are calculated at discrete points in space. However, during a motion, the hand moves through continuous space, where synergies are not necessarily calculated. To continue the feedback control process, the synergies at every single point in the task-space has to be identified. The synergies at an intermediate point can be estimated by interpolating between the mesh points.

A problem with the NNMF algorithm is its sensitivity to its initial guesses. In other words, there are very many local minima in the solution space, and the NNMF may hit a local minimum instead of the global minimum. Therefore, when the NNMF is run for two similar postures, the resulting synergies may be substantially different. These inconsistencies make the synergy interpolation between two adjacent points inapplicable.

Our approach to improve the consistency of NNMF is to use the synergies of a neighbouring point as the initial guess for the next posture. In this approach, the algorithm starts from one corner of the task-space with some random initial guess (see Fig. 4). Then the algorithm moves to the next point (posture) along the first dimension (the red line in Fig. 4), and uses the NNMF results of the previous point as its initial guess. The algorithm continuous to the end of the line, and uses the previous point's results as the initial guess. When the first line is covered, the algorithm moves along the second dimension (the blue lines in Fig. 4). The algorithm uses the NNMF results of the point in the previous line as its initial guess, until the first "page" (coloured in green) is covered. Finally, the algorithm moves along the last dimension, and uses the NNMF results of the points in the previous page as its initial guess, until the whole task-space is covered.

This algorithm results in more consistent synergy matrices across the postures. However, even after these efforts, there are occasional switches between columns of the synergy matrices (see Fig. 5a). To make sure that the order of the columns in the synergy matrices also remain consistent across different postures, we have used a k-mean clustering algorithm [30]. It identifies similar synergies, and then rearranges them in the right order in the synergy matrices. The combination of these two methods results in smooth transitions between synergies, which allows for easy interpolation.

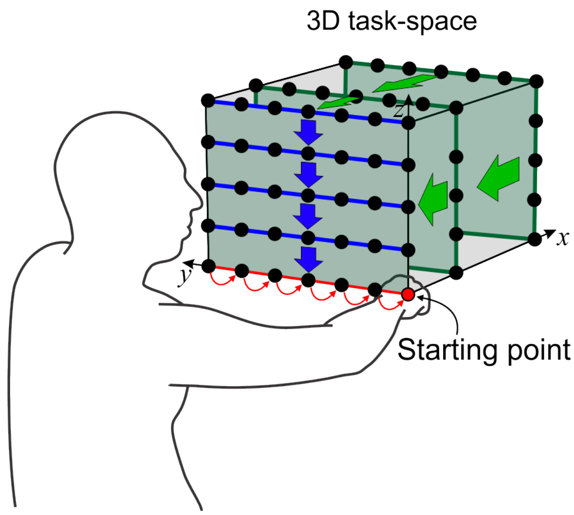


Fig. 4: The schematic of the progression through the task-space. The initial point is at the corner (the red dot). Each point along the first dimension (the red coloured line) uses the previous point results as the initial guess of the NNMF. Moving along the second dimension (the lines coloured blue), each point uses the result of the previous line as the initial guess. Finally, moving along the third dimension (the green "pages"), the previous page data is used as the initial guess of the current point.

2.2.2 Recalling and using synergies for feedback control—an on-line process

The pre-calculated (pre-learned) synergies can be used to quickly construct the muscle activations required in preforming an action. At a given posture, activation of a single synergy results in a certain task-space acceleration. Therefore, by activating all of the k synergies one by one, there will be k task-space acceleration vectors (Fig. 6). If the synergies are identified properly, these vectors will span the task-space (i.e., any arbitrary vector in the task-space can be written as a linear combination of these k vectors). Thus, the synergy-produced acceleration vectors make a *basis set* for the task-space.

Knowing the synergies and the corresponding basis set, it is possible to *positively-decompose* any arbitrary acceleration onto the basis set. In other words, one should solve for the k non-negative coefficients $\mathbf{C}_{k\times 1}$ such that:

$$\mathbf{a}_{ref} = \mathbf{B}_{3\times k}\mathbf{C}_{k\times 1} \quad , \quad c_i > 0 \tag{6}$$

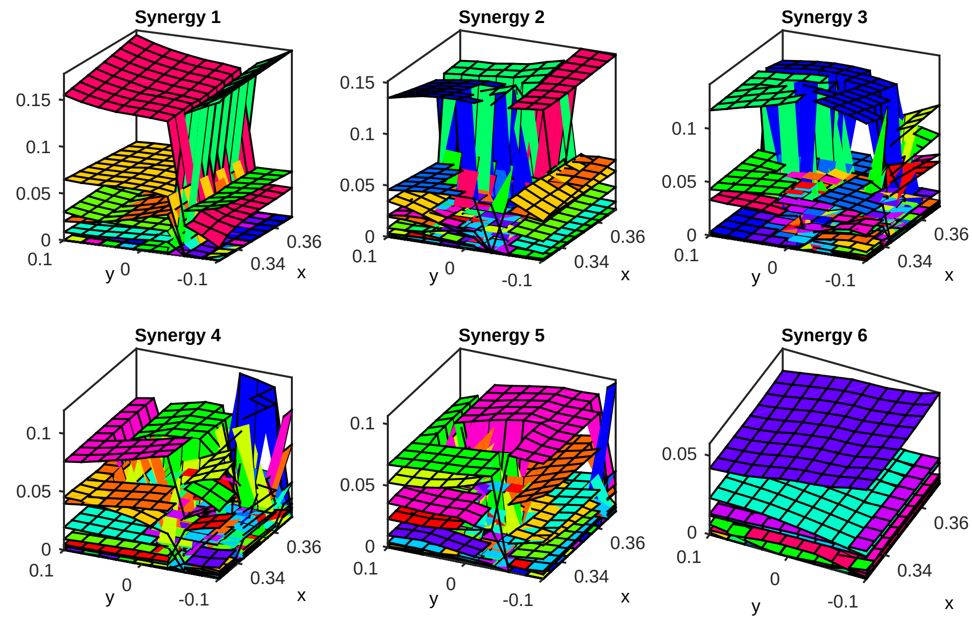
where $\mathbf{B}$ is the basis set matrix, whose columns are the 3D basis vectors. Although this problem does not necessarily have a unique solution, it can easily be solved using a least-squares method as implemented in `lsqnonneg` function in Matlab.

Unfortunately the human musculoskeletal system is a non-linear system, and one cannot use superposition. Muscle synergy theory relies on superposition where multiple synergies are combined. In general, the effect of the combination of multiple synergies is not the same as the summation of their individual effects. Fortunately, superposition is possible in certain cases. If we assume the body is stationary (no velocities) and neglect the elasticity of muscles, the relationship between the muscle activities and the resulting hand acceleration is linear, and superposition can be applied. In other words, the effect (total task-space acceleration) of co-activation of multiple synergies, is the same as the summation of the effects of individual synergies (which are the basis vectors).

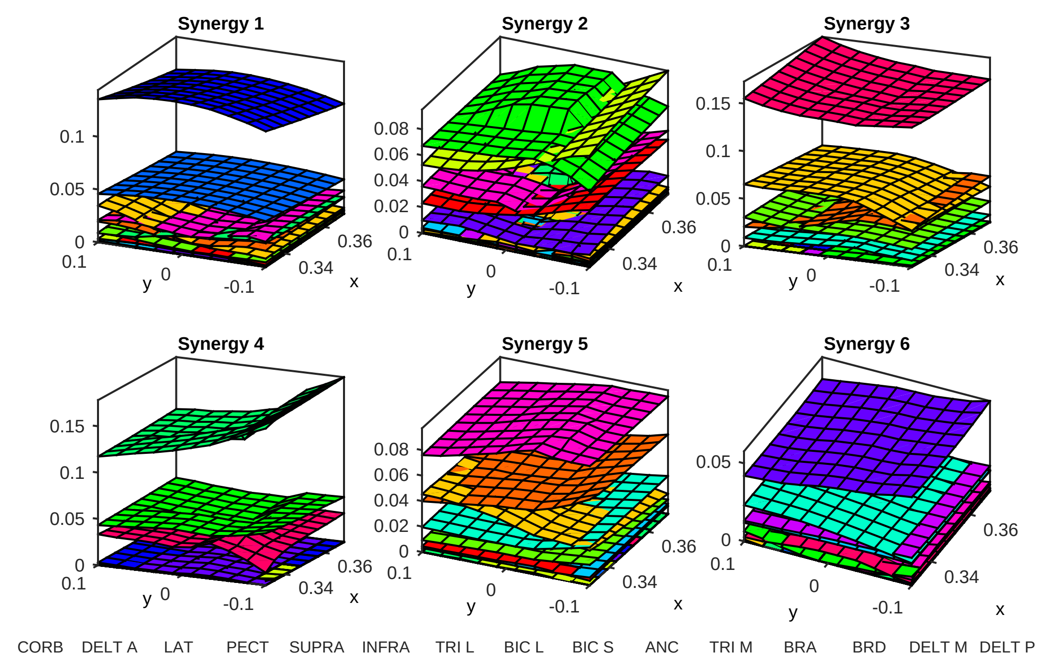
Thus, knowing how one acceleration vector can be created by linear combination of the basis vectors, it is possible to use the same coefficients to combine the corresponding synergies. This procedure is summarized in Fig. 6. In the example shown (with four synergies), the reference acceleration vector is decomposed into the basis set $\mathbf{B}$, resulting in non-zero coefficients c_1, c_2, c_3 and $c_4 = 0$. The coefficients multiplied by the synergy matrix give the activation vector:

$$\mathbf{u} = \mathbf{S} \times \mathbf{C} \tag{7}$$

The proposed acceleration-to-activation mapping has one important implication: we can replace the costly and time-consuming optimization process with a linear vector decomposition problem. Once the mapping from the



(a) Synergies before sorting



(b) Synergies after sorting with the clustering algorithm

Fig. 5: The pre- and post-sorting of the synergies. Each surface in the sub-figures shows the the share of a muscle (color coded) in one synergy. The plots show how the synergies change across postures (the *x* and *y* axes show the position of the hand, and vertical axis shows the synergies).

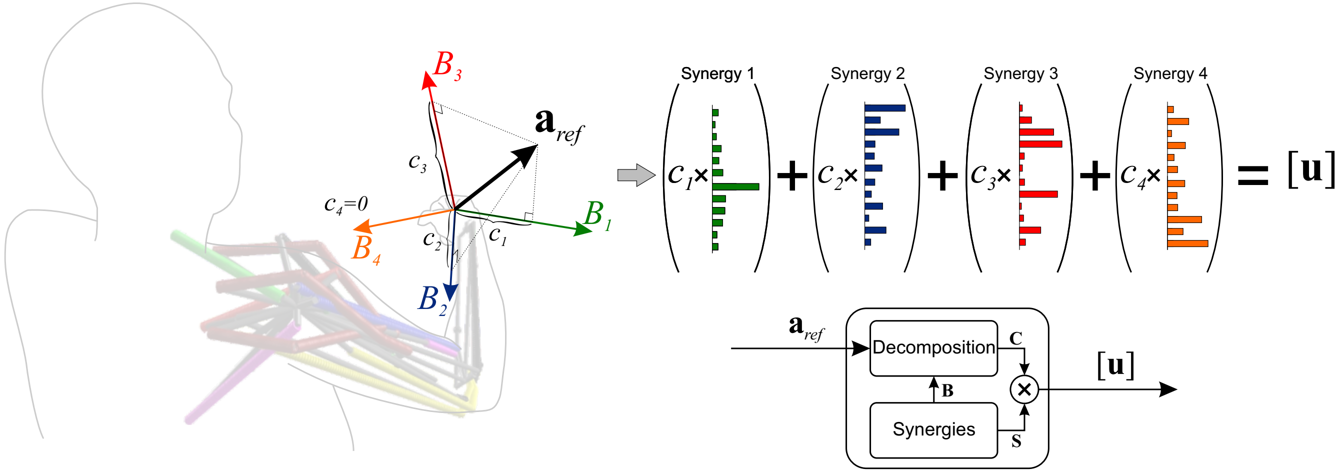


Fig. 6: The schematic of the acceleration-to-activation mapping. The same coefficients of the vector decomposition can be used to combine the synergies to produce the desired acceleration. Only four synergies are shown here to demonstration the concept.

desired acceleration to muscle activation is known, it can be used in the hierarchical scheme to control the motion (see Fig. 2).

Notes on the number of synergies: There is no systematic way to choose the number of synergies (except for the simpler cases described in [25]). The usual practice is to pick the smallest number of synergies that reconstruct the data (usually the experimental EMG) with little error. The reconstruction error usually decreases as the number of synergies increase, and the number of synergies beyond with no significant improvement is observed is chosen.

However, the goal in our framework is the optimality of the whole control process, instead of the reconstruction error of the original data (i.e. the error in (5)). Therefore, we have considered a different measure to choose the number of synergies. the *sub-optimality* of acceleration-to-activation mapping is chosen as the measure. The sub-optimality is defined as:

$$\text{sub-optimality} = \frac{|\mathbf{u}_{syn} - \mathbf{u}_{opt}|}{|\mathbf{u}_{opt}|} \tag{8}$$

where $\mathbf{u}_{syn}$ is the vector of muscle activations calculated from the synergy-based acceleration-to-activation mapping, and $\mathbf{u}_{opt}$ is the optimally calculated one.

2.2.3 Taking the system dynamics into account

It is important to note that the acceleration-to-activation mapping is calculated for a stationary condition. For a better control performance, velocity-dependent accelerations, gravity, and external forces must be accounted for in the calculations.

The arm dynamics in the joint-space can be described as:

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{b}(\mathbf{q},\dot{\mathbf{q}}) + \mathbf{g}(\mathbf{q}) = \mathbf{J}^T\mathbf{F} + \mathbf{f}(\mathbf{u}) \tag{9}$$

where $\mathbf{q}$ is the vector of the joint angles, $\mathbf{M}$ is the inertia matrix, $\mathbf{b}$ contains the velocity dependent terms, and $\mathbf{g}$ represents the effect of gravity. $\mathbf{F}$ is the vector of the external forces on the hand, and $\mathbf{f}(\mathbf{u})$ represent the effect of muscle forces on the system. The Jacobian matrix $\mathbf{J}$ relates the task-space (hand position) to joint-space.

In the nominal condition (no gravity, no external forces, no velocity), the muscle activations found from the acceleration-to-activation mapping ($\mathbf{u} = \mathscr{T}(\mathbf{a}_{ref})$) result in the reference acceleration. In joint space representation:

$$\ddot{\mathbf{q}}_{ref} = \mathbf{M}^{-1}\mathbf{f}(\mathscr{T}(\mathbf{a}_{ref})) \tag{10}$$

However, in a general condition where the non-linear terms exist, the resulting acceleration will be different from the desired one.

$$\underbrace{\ddot{\mathbf{q}}_{real} + \mathbf{M}^{-1}(\mathbf{b} + \mathbf{g} - \mathbf{J}^T\mathbf{F})}_{\tilde{\ddot{\mathbf{q}}}_{ref}} = \mathbf{M}^{-1}\mathbf{f}(\mathscr{T}(\mathbf{a}_{ref})) \quad (11)$$

$$\tilde{\ddot{\mathbf{q}}}_{ref} = \mathbf{M}^{-1}\mathbf{f}(\mathscr{T}(\tilde{\mathbf{a}}_{ref})) \quad (12)$$

Therefore, in order to have $\ddot{\mathbf{q}}_{real} = \ddot{\mathbf{q}}_{ref}$, one should use the augmented acceleration in the mapping, i.e.:

$$\tilde{\ddot{\mathbf{q}}}_{ref} = \ddot{\mathbf{q}}_{ref} + \mathbf{M}^{-1}(\mathbf{b} + \mathbf{g} - \mathbf{J}^T\mathbf{F}) \quad (13)$$

Since the task-space acceleration can be written as:

$$\mathbf{a} = \mathbf{J}\ddot{\mathbf{q}} + \dot{\mathbf{J}}\dot{\mathbf{q}} \quad (14)$$

the augmented acceleration in the task-space will be:

$$\begin{aligned} \tilde{\mathbf{a}}_{ref} &= \mathbf{J}\tilde{\ddot{\mathbf{q}}}_{ref} + \dot{\mathbf{J}}\dot{\mathbf{q}} && (15)\\ &= \mathbf{J}(\ddot{\mathbf{q}}_{ref} + \mathbf{M}^{-1}(\mathbf{b} + \mathbf{g} - \mathbf{J}^T\mathbf{F})) + \dot{\mathbf{J}}\dot{\mathbf{q}} && (16)\\ &= \mathbf{a}_{ref} + \mathbf{J}\mathbf{M}^{-1}(\mathbf{b} + \mathbf{g} - \mathbf{J}^T\mathbf{F}) + \dot{\mathbf{J}}\dot{\mathbf{q}} && (17) \end{aligned}$$

3 Results

3.1 Number of synergies

The effect of number of synergies on the sub-optimality is shown in Fig. 7. Because the NNMF is sensitive to the initial guess, each run of the NNMF may result in a different synergy matrix, and consequently a different sup-optimality. Therefore, multiple runs of NNMF are performed (50 runs for each number of synergies in Fig. 7). It can be seen in Fig. 7a that by increasing the number of synergies, the reconstruction error decreases until $k = 10$, after which it starts to increases again. When 15 synergies are used, the reconstruction error is negligible, as each synergy will include only one muscle.

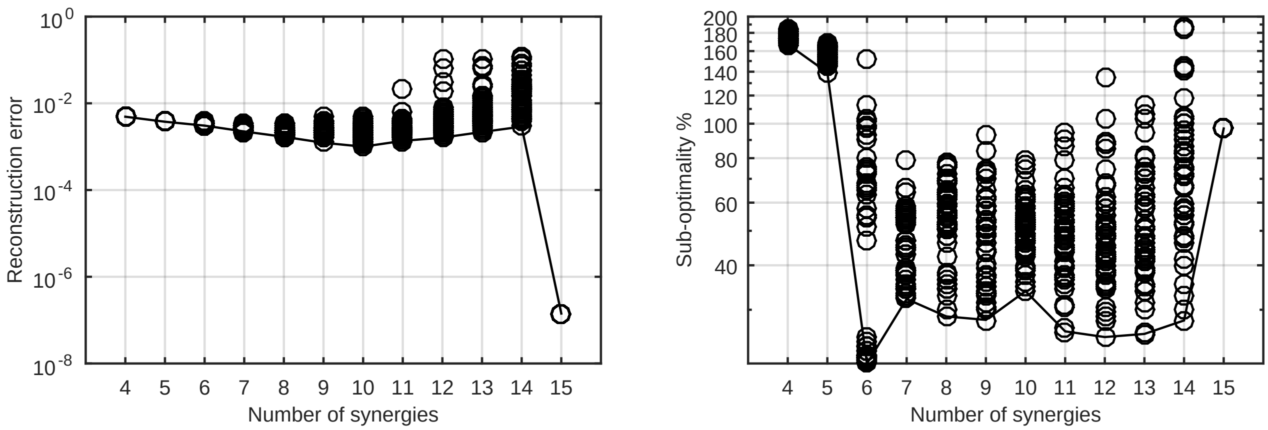
The sub-optimality measure is shown in Fig. 7b. Low number of synergies (four and five) result in large sub-optimality as expected. Counter-intuitively, we observe that increasing the number of synergies beyond six does not reduce sub-optimality. Therefore, for the next set of simulations, six synergies has been used.

3.2 Closed loop controller performance

The closed-loop control performance of the proposed motor control framework is shown in Fig. 8. The desired motion is a 20 cm periodic motion in y (medial-lateral) direction shown in Fig. 8a. Two control methods are used to track the motion: the proposed framework, and a nonlinear model predictive controller (in figures denoted by “synergy” and “optimal”, respectively). The tracking error obtained from the two controllers are shown in Fig. 8a, with the muscle activations presented in Fig. 8b. The performance of the two controllers are similar in tracking error, and the muscle activations are also similar. The total physiological effort (defined as: $\int_0^{t_f} \mathbf{u}^T\mathbf{u}\, dt$) and the CPU time of the two controllers are given in Table 1.

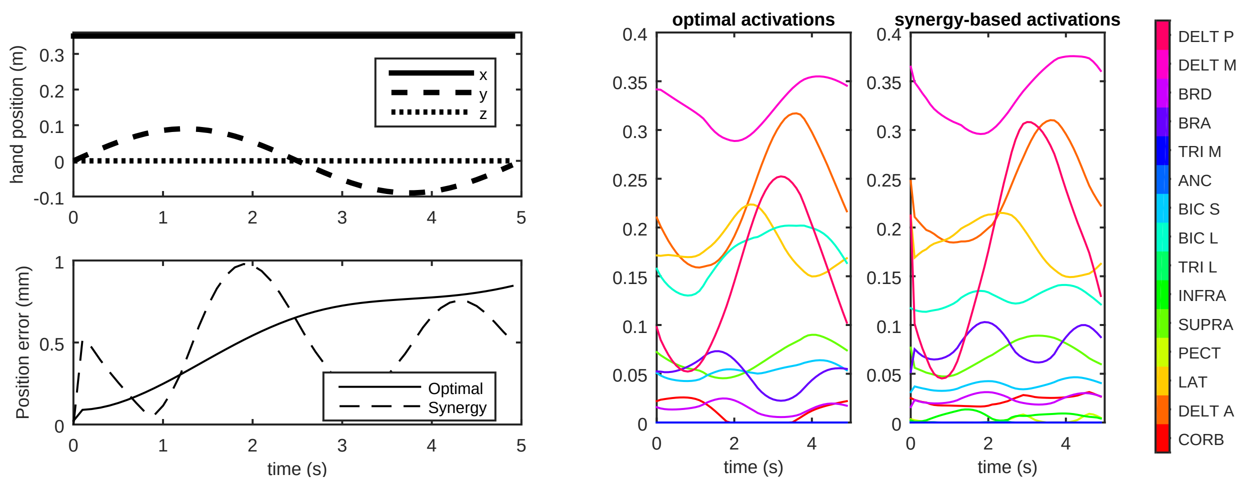
4 Discussion

The proposed motor control framework is based on a hierarchical structure, where the task-space control is separated from low-level *muscle-space* control. The bridge between the task-space and muscle-space control signals is a posture-dependent mapping; it translates the task-space acceleration signal to muscle activations.



(a) (b)

Fig. 7: The effect of number of synergies on (a) the NNMF reconstruction error, and (b) sub-optimality of acceleration-to-activation mapping



(a) (b)

Fig. 8: The closed-loop controller performance.

Control method	Physiological effort	CPU time
Optimal	0.1138	103 s
Synergy	0.1165 (2% increase)	8.4 s (92% decrease)

Tab. 1: The physiological effort of the two controllers

The task-space controller chosen in this paper was a simple error-driven PID controller. Although the PID showed acceptable results in the simulations, it may not capture the important properties of the human motor control system, e.g., learning and adaptation. For this purpose, more complex controllers (e.g. artificial neural networks, or model-based controllers) may better represent the nervous system.

The acceleration-to-activation mapping introduced in this paper employed muscle synergies to simplify the calculations and the processing time. It was also assumed that an internal representation of the arm (through the known basis vectors associated with each synergy) is available to the nervous system.

The number of the synergies is still an uncertain matter in the literature. In this work the new concept of *sub-optimality* of the acceleration-to-activation mapping was introduced, and used to decide on the number of synergies. The data reproduction capability measures (e.g. the variance accounted for, VAF) that has been in use in the literature has one problem: it constantly decreases by increasing the number of synergies. Thus, one has to decide how much improvement is enough.

The sub-optimality measure is at minimum with six synergies. The reason can be summarized as follow. With low number of synergies (four and five), there is not enough synergies to capture all the variation in the optimal data; therefore, the the mapping results will be sub-optimal. As the number of synergies increase, more data variation can be captured. However, with high number of synergies, the NNMF tends to put a single muscle in each synergy to maximize the degree of freedom. Therefore, when the basis vectors of the synergies are used to produce an arbitrary acceleration vector, only a few muscles in a non-optimal way are combined, resulting in a higher sub-optimality.

5 Conclusions

We presented a motor control framework to control motion of a musculoskeletal system in 3D space. The hierarchical structure of the models allows for fast feedback control, by separating the feedback control of the task-space variables from the complexities of the musculoskeletal system. The bridge between the task and muscle spaces is a mapping that converts the acceleration signals in the task-space to muscle activation. This mapping uses posture-dependent muscle synergies and the corresponding basis vectors for the acceleration decomposition. The simulation results show strong potential of the proposed motor control framework for the optimal feedback control of musculoskeletal systems, which can replace the time-consuming optimization procedures.

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